Acceleration to Terminal Velocity of Cloud and Raindrops

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ABSTRACT

A theoretical method is given which allows computing the acceleration to terminal velocity of cloud and raindrops at various levels in the atmosphere. For drops of equivalent radius 800 μm $\leqslant a_0 \leqslant$ 3500 μm our theoretical predictions were found to agree well with the results of an experimental study carried out in the UCLA Rain-Shaft. For drops of 20 μm $\leqslant a_0 \leqslant$ 80 μm our theoretical predictions were found to agree well with the experimental results of Sartor and Abbott (1975). Experiment and theory indicate that in air of 1000 mb and 20°C, drops of $a_0 >$ 1000 μm need distances of at least 12 m to accelerate to terminal velocity.

1. Introduction

For a realistic simulation in the laboratory of the hydrodynamic behavior of cloud and raindrops it is necessary to design cloud chambers in which the drops are able to reach terminal velocity. For this reason numerous researchers have taken great pains to design wind tunnels in which the drops are freely supported in an air stream (Blanchard, 1950; Komabayasi *et al.*, 1964; Cotton and Gokhale, 1967; Spengler and Gokhale, 1970; Pruppacher and Neiburger, 1968; Beard and Pruppacher, 1969; Montgomery and Dawson, 1969), or constructed acceleration systems through which millimeter size drops are accelerated over relative short distances to terminal velocity (McTaggert-Cowan and List, 1975a, b).

Many times, however, researchers find themselves in a situation where sophisticated experimental equipment, such as expensive wind tunnels, are not available so that they must design a more simple chamber. Such construction requires knowing the distance over which the water drops, planned to be studied, reach terminal velocity. Except for some older field experiments by Laws (1941), who studied the acceleration of millimeter size drops under rather unsatisfactorily controlled external conditions, and some recent laboratory work by Sartor and Abbott (1975) with small drops of radii of 20 to 80 μm, little is known about the distance water drops in air require to accelerate from rest to terminal velocity. Also no theory exists with which one would be able to theoretically predict this distance for any temperature and pressure of the air in which the water drops fall.

Recent literature shows that knowing the acceleration distance of drops to terminal velocity is beneficial not only to experimentation in cloud microphysics but also to studies designed to determine the efficiency with which cloud and raindrops scavenge air pollutants. Thus, Starr and Mason (1966) studied the scavenging efficiency of water drops up to 1 mm (100 μm) in radius inside a 1.2 m high chamber; Hampl *et al.* (1971), Hampl and Kerker (1972) and Kerker and Hampl (1974) made scavenging studies with drops up to 2.5 mm in radius inside cloud chambers up to 5.15 m height; and Adam and Semonin (1970) investigated the collection efficiency of water drops of 1.25 mm (1250 μm) radius for submicron aerosol particles in a shaft of 12 m height. The experimental acceleration studies of Laws (1941), which show that drops between 1 and 3 mm equivalent radius require distances between 12 and 17 m for acceleration to terminal velocity, suggest that the above mentioned scavenging studies were carried out in chambers in which the drops did not at all, or only marginally, reach terminal velocity.

In order to shed some further light on this problem we carried out a drop-acceleration experiment inside the UCLA Rain-Shaft. We also extended the theoretical method of Beard (1976) for predicting drop terminal velocities to include predicting the time and distance water drops need to reach terminal velocity in air of specified temperature and pressure.

2. Experimental set-up

The present experiments were carried out inside the UCLA Rain-Shaft. This facility was erected during the construction of the Mathematical Sciences building of UCLA in 1968. The shaft extends vertically over nine building floors, measures 35 m in height and has a cross section of about 1 m^2. In order to confine an air volume whose temperature, humidity and purity could be controlled, a cylindrical plexiglass tube of 0.45 m in diameter was inserted into the shaft. The tube consists of 3 m long sections, tightly sealed against each other. The top and bottom of this plexiglass shaft can be connected to a closed-circuit air circulation system inside of which the air is circulated by means of a pump through air driers and air filters to achieve the desired uniformity of temperature, humidity and air purity. The shaft connects one small laboratory at the top and one at the bottom of the shaft. The temperature of each of these laboratories can be set by means of independently controllable thermostats. In order to avoid overturning of the air in the shaft the temperature of the air at the bottom of the shaft was set at about 2°C colder than the temperature of the air at the top of the shaft. The water drops whose acceleration to terminal velocity was studied had equivalent radii of 850, 1350, 2000, 2500 and 3350 μm. These drops were produced by a specially designed technique involving a hypodermic needle and a constant head of pure water. The equivalent radius a_0 (which is the radius of a spherical drop with the same volume as the deformed drop) of each drop was determined from the volume of water necessary to form it, and independently by direct measurement allowing the drop to fall into a dish filled with silicone oil of a viscosity which prevented the drop from sinking significantly. Using these size determinations it was also established that the size change of the drops while falling through the water-subsaturated air column in the shaft was less than 1%.

To determine a drop's acceleration each drop was electrically charged by an induction method after release just before entering the shaft. Subsequently, the drop was allowed to fall through 21 induction rings equally spaced at 1.00 m. As the electrically charged drops fell through the induction rings connected to an electrometer, sharply defined pulses were registered by a fast pen recorder. From a precise knowledge of the chart speed of this recorder and the distance between the electrometer rings the mean fall velocity of a drop between a pair of rings was determined. Considering these velocities, the drop's time and distance to reach 99.1% of its terminal velocity was computed.

3. Theory

The equation of motion of a water drop of mass m is

$$m\frac{dV}{dt}=mg\frac{(\rho_w-\rho)}{\rho_w}-F_D=mg\left(\frac{\rho_w-\rho}{\rho_w}\right)-6\pi a\eta V\left(\frac{C_D\mathrm{N_{Re}}}{24}\right), \quad (1)$$

where V is the instantaneous velocity of the drop, t time, a the radius of the drop, ρ_w the density of water, η and ρ are the dynamic viscosity and density of air, respectively, F_D is the drag force on the drop, g is the acceleration of gravity, $C_D=2F_D/\pi a^2\rho V^2$ is the drag force coefficient of the drop, and $\mathrm{N_{Re}}=2a\rho V/\eta$ is the Reynolds number of the drop. In general, this equation must be solved numerically since both C_D and $\mathrm{N_{Re}}$ are functions of V. However, an approximate analytical solution may be obtained for very small drops falling in the Stokes regime. Then $C_D\mathrm{N_{Re}}/24=1$ and the resulting linear equation may be immediately integrated to obtain

$$V(t)=V_\infty\left[1-\exp\left(-\frac{6\pi a\eta}{m}t\right)\right], \quad (2)$$

where V_∞ is the drop's terminal velocity. Thus, the viscous relaxation time τ_S for a spherical drop falling in Stokes regime is

$$\tau_S=\frac{m}{6\pi a\eta}=\frac{2a^2\rho_w}{9\eta}\approx 1.2\times10^3a^2 \quad (3)$$

for a in cm, and with $\eta=1.818\times10^{-4}$ poise (1.818×10^{-5} kg m^{-1} s^{-1}) at 20°C. Similarly, the time required for the drop to reach a fraction β of its terminal velocity is seen to be

$$t_\beta=-\tau_S\ln(1-\beta), \quad (4)$$

$$t_{99\%}=1.02(a^2\rho_w/\eta). \quad (5)$$

Integration of (2) gives for the distance z_β the drop must fall to reach a fraction β of its terminal velocity the result

$$z_\beta=-\frac{mV_\infty}{6\pi a\eta}[\ln(1-\beta)+\beta]. \quad (6)$$

For example,

$$z_{99\%}=0.70(a^2\rho_wV_\infty/\eta). \quad (7)$$

Thus, for a 30 μm radius drop falling in air at 20°C, $z_{99\%}\approx5000$ μm or about 170 drop radii. These simple computations show that drops <30 μm obviously need only very short distances to reach terminal velocity. It shall be the purpose of the remainder of this paper to show that $z_{99\%}$ quickly increases with increasing drop size to above 10 m for millimeter size water drops, i.e., distances which cannot be accommodated in cloud chambers located inside normal sized laboratories.

In order to determine the fall velocity as a function of time for water drops of size outside the Stokes regime accelerating under gravity in air we need to consider the complete equation (1). After introducing the defining equation for the Reynolds number we may rewrite (1) in the form

$$\frac{d\mathrm{N_{Re}}}{dz}=\frac{\left[g\left(\frac{\rho_w-\rho}{\rho_w}\right)-\frac{3}{32}(C_D\mathrm{N_{Re}^2})\frac{\eta^2}{\rho\rho_wa_0^3}\right]\left(\frac{2\rho a_0}{\eta}\right)^2}{\mathrm{N_{Re}}}, \quad (8)$$

where $c_D\mathrm{N_{Re}^2}$ is a function of the Reynolds number and drop shape.

Beard (1975) has shown that for spherical water drops at terminal velocity in air, and of sizes $9.5 \leqslant a_0 \leqslant 535$ μm, $c_D N_{Re}^2$ is semi-empirically related to the Reynolds number by

$$N_{Re} = \exp(Y), \tag{9}$$

where

$$Y = B_0 + B_1 X + \cdots + B_6 X^6, \tag{10}$$

with

$$B_0 = -0.318657 \times 10^1, \quad B_1 = 0.992696,$$
$$B_2 = -0.153193 \times 10^{-2}, \quad B_3 = -0.987059 \times 10^{-3},$$
$$B_4 = -0.578878 \times 10^{-3}, \quad B_5 = 0.855176 \times 10^{-4},$$
$$B_6 = -0.327815 \times 10^{-5},$$

and with

$$X = \ln(C_D N_{Re}^2). \tag{11}$$

For deformed drops at terminal velocity, and sizes $535 \leqslant a_0 \leqslant 3500$ μm, Beard (1975) showed that $C_D N_{Re}^2$ can be obtained from the semi-empirical relations

$$N_{Re} = N_P^{\frac{1}{6}} \exp(Y), \tag{12}$$

where

$$Y = B_0 + B_1 X + \cdots + B_5 X^5, \tag{13}$$

with

$$B_0 = -0.500015 \times 10^1, \quad B_1 = 0.523778 \times 10^1,$$
$$B_2 = -0.204914 \times 10^1, \quad B_3 = 0.475294,$$
$$B_4 = -0.542819 \times 10^{-1}, \quad B_5 = 0.238449 \times 10^{-2},$$

and with

$$N_P = \frac{\sigma^3 \rho^2}{\eta^4 (\rho_w - \rho) g}, \quad X = \ln(3 N_{Bo} N_P^{\frac{1}{6}}/4), \tag{14}$$

where σ is the surface tension of water against air. For a drop at terminal velocity, the physical property number N_p is related to the Davies number (also sometimes called the Best number)

$$N_{Da} \equiv c_D N_{Re}^2 - 32 a_0^3 \rho (\rho_w - \rho) g / 3\eta^2$$

and to the Bond number $N_{Bo} = 4a_0^2(\rho_w - \rho) g/\sigma$ by the relation

$$C_D N_{Re}^2 = 4(N_p N_{Bo}^3)^{\frac{1}{2}}/3 = 3^{\frac{1}{2}}(N_P^{\frac{1}{2}} e^{3X})^{\frac{1}{2}}/2 \tag{15}$$

by using $F_D = mg(\rho_w - \rho)/\rho_w$ with the defining relation for C_D.

As long as the water drop is sufficiently small such that it remains spherical at all times, i.e., as long as $C_D = C_D(N_{Re})$ only, Beard's relations [(9)–(11)] may be used unaltered to determine the instantaneous Reynold's number N_{Re}, i.e., the instantaneous velocity V of the drop as a function of its distance z after release from rest. Thus for a given temperature and pressure of the atmosphere which determines ρ_w, ρ and η, and for a given equivalent drop size a_0, Eqs. (8)–(11) constitute a closed set from which $N_{Re}(z)$, and from which

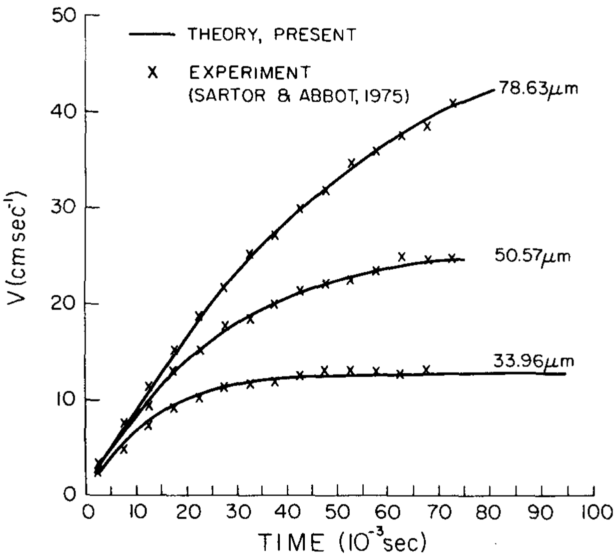

FIG. 1. Variation of fall velocity with time elapsed after release from rest of water drops in air at 828 mb and 22°C—comparison of present theory with experiments of Sartor and Abbott (1975) for small, spherical drops.

because of

$$V = \frac{N_{Re} \eta}{2 a_0 \rho}, \tag{16}$$

$V(z)$ of the drop can be calculated.

The problem of determining the instantaneous velocity of an accelerating drop becomes considerably more difficult if the drop is deformed. For such a case the Beard (1975) relations (12)–(15) may not be applicable without making further assumptions. This becomes necessary since $C_D = C_D(N_{Re}, \text{shape})$ which implies that obtaining an instantaneous C_D requires knowing the instantaneous shape of the drop. The latter, however, is not known. On the other hand, it is physically reasonable to assume that the instantaneous C_D of a deformed drop of radius a_0 can be approximated roughly by C_D of that smaller drop at terminal velocity which has a Reynolds number equal to the instantaneous Reynolds number of the larger drop, i.e., C_D of an accelerating drop is found from the C_D curve for drops at terminal velocity at the same N_{Re}; such a C_D curve for drops at terminal velocity is implied by the formulae found in Beard (1975). Through this assumption it now becomes possible to describe the acceleration behavior of a drop of radius a_0 by equations which apply to a smaller drop at terminal velocity. Eqs. (8) and (12)–(15) are then again a closed set from which for given ρ_w, ρ and η for a given drop of radius a_0, one can compute $N_{Re}(z)$, and because of (16) one can compute $V(z)$ of that drop.

4. Results and discussion

The theoretically computed acceleration distances for spherical drops are compared in Fig. 1 with selected

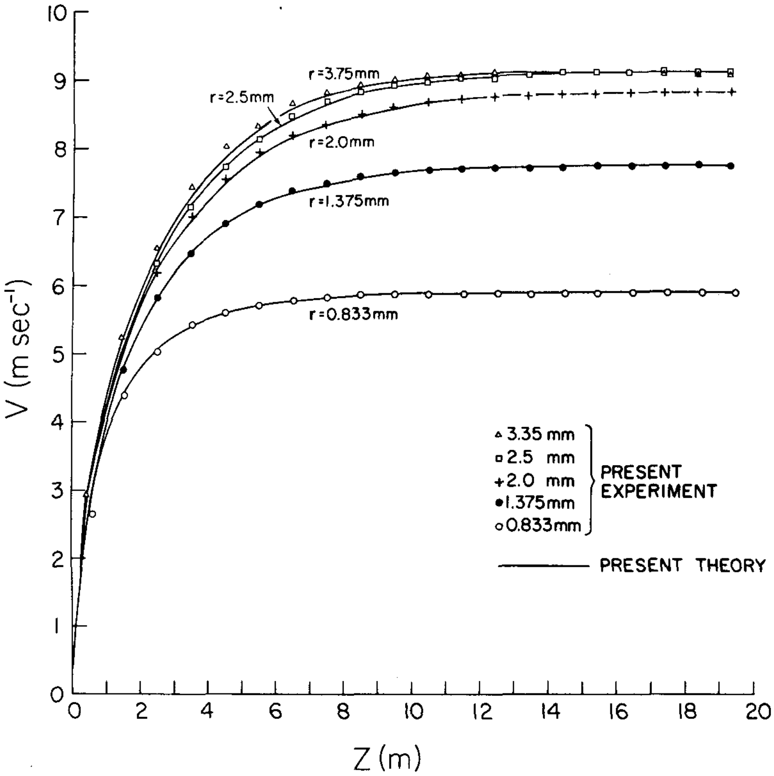

FIG. 2. Variation of fall velocity with distance after release from rest of water drops in air at 1000 mb and 20°C—comparison of present theory with experiments for large, deformed drops. Note: top most curve should be labeled $r=3.35$ mm.

experimental results for spherical drops obtained by Sartor and Abbott (1975). It is seen that the present theory and experiment agree well. Good agreement also is found (not shown here) between the present theory and Sartor and Abbott's (1975) semi-empirical analytical formulations based on the LeClair *et al.* (1970) values for the drag on water drops in air.

In Fig. 2 comparison is made between the present

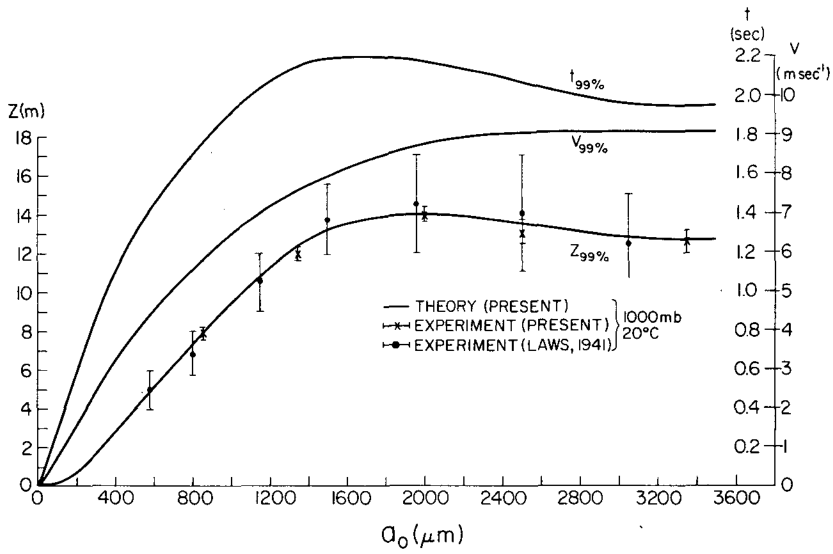

FIG. 3. Variation of drop velocity $V_{99\%}=0.99\ V_{\infty}$, fall time $t_{99\%}$ to reach $V_{99\%}$ and fall distance $z_{99\%}$ to reach $V_{99\%}$, with equivalent drop radius a_0—comparison of present theory with present experiments and experiments of Laws (1941) at 1000 mb, 20°C.

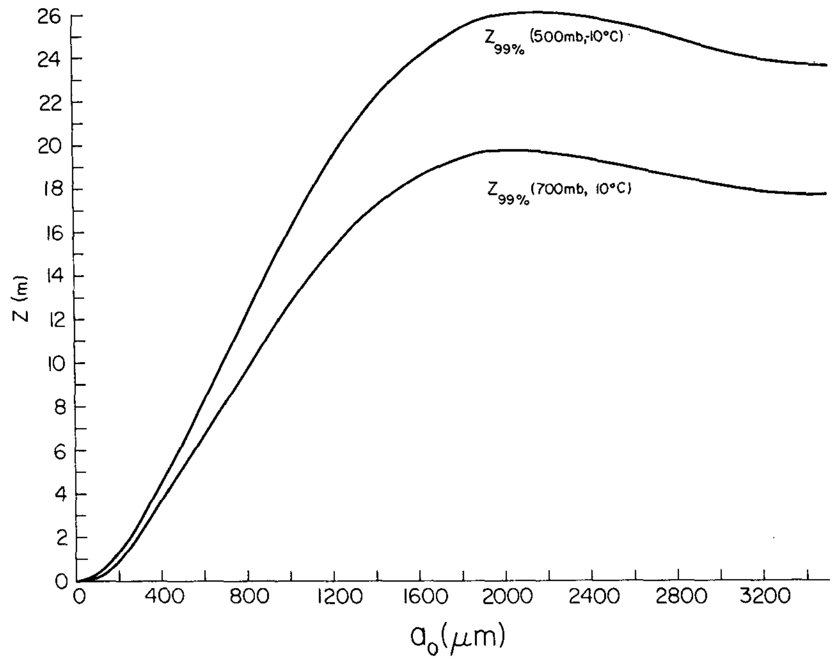

FIG. 4. Variation of fall distance $z_{99\%}$ to reach $V_{99\%}=0.99\ V_\infty$ as a function of equivalent drop radius a_0—present theory for 700 mb, 10°C and 500 mb, −10°C.

theoretical predictions for deformed drops. Again excellent agreement is found between the theory and experiment thus justifying the assumptions underlying the present theory. In Fig. 3 the distances $z_{99\%}$ experimentally determined by us and by Laws (1941) are compared with our theoretical predictions. Again agreement between experiment and theory is demonstrated.

The good agreement between experiment and theory motivated us to compute $z_{99\%}$ as a function of a_0 for pressure levels and temperatures other than 1000 mb and 20°C. The results of these calculations are displayed in Fig. 4. Thus, for 700 mb and 10°C, and for a_0= 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000 μm, $z_{99\%}$=0.18, 0.90, 2.1, 3.6, 5.4, 6.7, 8.2, 9.6, 11.1, 12.6, 19.8, 18.1 m, respectively. For 1000 mb and 20°C, $z_{99\%}$ for the same size drops is 0.15, 0.71, 1.6, 2.8, 3.9, 5.2, 6.3, 7.3, 8.4, 9.5, 14.0, 12.8 m. The above results clearly show that drops with radii $\gtrsim$500 μm require distances for acceleration to terminal velocity which cannot be accommodated in cloud chambers located inside normal size laboratories.

Note from Fig. 3 that the time $t_{99\%}$ for a drop to reach 99% of its terminal velocity rapidly increases with increasing drop size, reaches a maximum near 1600 μm and subsequently decreases slowly with further increase in a_0. Since $z_{99\%}=V_{99\%}\times t_{99\%}$ and since $V_{99\%}$ continuously increases with increasing drop size to level off near 2400 μm, $z_{99\%}$ exhibits a maximum which is shifted, to about 2000 μm radius. This maximum, although already evident in the early experimental data of Laws (1941), has clearly been identified by the present experimental results and is independently predicted by the present semi-empirical model. The maximum can easily be understood if we consider that an increase in drop size results not only in an increased drop mass but also in an increased drop deformation. Thus, as a_0 increases in the size range where the drop still remains relatively spherical in shape, i.e., $a_0\lesssim$500 μm (Pruppacher and Beard, 1970; Pruppacher and Pitter, 1971), $t_{99\%}$ increases due to the increased mass of the drop. However, at a_0>500 μm the amount a drop is deformed at terminal velocity rapidly increases with increasing size (Pruppacher and Pitter, 1971; Pruppacher and Beard, 1970). As a result the drop becomes deformed earlier and earlier during a drop's acceleration to terminal velocity. Since increased deformation increases the drag force coefficient of a drop it effectively acts as a "brake" on the drop's fall velocity, finally overcompensating the increase in mass by the increase in hydrodynamic drag. It is of course obvious that at lower pressure levels in the atmosphere $t_{99\%}$, i.e., $z_{99\%}$, will be larger than at 1000 mb, and that the maximum in $t_{99\%}$ and $z_{99\%}$ shifts to values of a_0 which are larger than those at the 1000 mb pressure level. This is indicated in Fig. 4.

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