# A Wind Tunnel Investigation of the Effect of an External, Vertical Electric Field on the Shape of Electrically Uncharged Rain Drops

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ABSTRACT

Results are presented of a recent wind tunnel experiment in which electrically uncharged water drops of 1000–3000 $\mu$m equivalent radius were freely suspended in the vertical air stream of the UCLA Cloud Tunnel. During their suspension, the drops were exposed to external, vertical electric fields of 0–9000 volts cm$^{-1}$. The change in the drop shape with drop size and with electric field strength was noted and is discussed in the light of theoretical work cited in literature which does not take into account the feedback effects between the electric forces of an external electric field and the hydrodynamic forces due to the flow past the drop. In contrast, the present wind tunnel study, documented by photographs from a 16 mm motion picture film, recorded the shape of the water drop in response to *both* hydrodynamic as well as electric forces.

## 1. Introduction

Precipitation from clouds strongly affects the propagation of electromagnetic radiation. In the recent past, much attention has been given to the determination of the level of dependence of the scattering of electromagnetic radiation and of wave lengths as used by present day radar systems on the size and the number concentration of precipitation particles. In particular, recent radar studies show that much progress has been made in determining size distribution of rain drops and rainfall rates by radar, based on the knowledge of a relation between raindrop shape and size (see e.g., Seliga and Bringi, 1976; Seliga *et al.*, 1981).

Thus far, radar meteorologists have used for their rainfall rate computation the drop shapes theoretically predicted by Pruppacher and Pitter (1971) and verified by the wind tunnel studies and those of Pruppacher and Beard (1970). These drop shapes were computed under the assumption that surface tension and hydrostatic and aerodynamic forces are in equilibrium, and are the only acting forces. In addition it was assumed that the drops were neither oscillating nor electrically charged and that no external electric fields were present.

In actuality, however, drops in the free atmosphere undergo complicated oscillations in response to atmospheric turbulent motions, to collision and coalescence between the drops, and to the shedding of air vortices from their downstream side. Furthermore, the drops in many clouds are already electrically charged at an early stage of the cloud life cycle and are exposed to the external electric fields present in the electrically charged clouds. Each of these factors profoundly affects the drop shape. In the present report we concern ourselves *exclusively* with the effect of an external, vertical electric field on the drop shape.

## 2. Critical review of previous work on effect of external electric fields on drop shape

Although several theoretical studies have been reported in the literature on the effect of an external elec-

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tric field on the shape of a drop of liquid, to date no theory exists which would take into account the complete problem involving the feedback effects between the electric forces on the drop due to the external electric field, and the aerodynamic forces due to the flow inside and around a drop falling at terminal velocity.

Instead, O'Konski and Thatcher (1953), Taylor (1964), Brazier-Smith (1971a,b) and Abbas and Latham (1969) theoretically determined the deformation, as given by $b/a$ (where $a$ is the drop's semiaxis perpendicular to the drop's fall axis, and $b$ is the drop's semiaxis along the drop's fall axis) of a *static* drop of originally spherical shape. These studies show that an originally spherical water drop becomes increasingly deformed (as a prolate spheroid) under the influence of an external electric field. The drop deformations predicted by these theories appear to agree quite well with each other, in particular regarding the maximum stable drop deformation beyond which the drop will break up. This fact is illustrated by Fig. 1 in which the theoretically predicted shape of a water drop in air is plotted as a function of the nondimensional quantity $X = E_0(a_0/\sigma_{wv})^{1/2}$, where $E_0$ is the field strength of the external electric field, $a_0$ the equivalent drop radius and $\sigma_{wv}$ the surface tension of water. The nondimensional quantity $X$ has been introduced on the basis of dimensional arguments by Zeleny (1915, 1917) and Taylor (1964), who considered that the parameter involved in the *critical condition* for mechanical stability must be based on the equality of the surface electrostatic stresses and the surface tension stresses. For *a sphere,* the first is given by $E_0^2/8\pi$ and the second by $2\sigma_{wv}/a_0$. Thus, on equating the two, one finds that $E_{0c}(a_0/\sigma_{wv})^{1/2}$ = constant where $E_{0c}$ indicates the critical electric field at the onset of drop instability. In the case of a real drop, the geometric form at the onset of instability is different from that of a sphere. Computations for *real drops* give a dimensionless constant $X_c = E_{0c}(a_0/\sigma_{wv})^{1/2}$, where the subscript $c$ indicates the critically unstable condition. Taylor (1964) predicted $X_c = 1.625$ while Brazier-Smith (1971b) found $X_c = 1.603$, with the corresponding values $b/a = 1.86$ and 1.83 respectively. These theoretical results shown in Fig. 1 suffer from the fact that they do not include the effects of aerodynamic forces due to flow around and inside the drop.

Drop breakup was also addressed by Richards and Dawson (1973) who determined for three observed drop shapes (photographed in the presence of an external electric field) the stresses on the drop falling at terminal velocity in air; from these they deduced the aerodynamic pressure profiles around the drop. Finally, Zrink (1984) obtained theoretical solutions for the relation between the eccentricity of raindrops and the electric charge on the drop and the strength of an external electric field, using the model of Green (1975) for describing the drop shape. Zrink deduced from this theory that the external electric fields which would be required for affecting the drop shape are considerably larger than those commonly observed in the atmosphere; however, electric charges which typically may reside on the drops in clouds enhance the oblateness of the drop. Unfortunately, Zrink based his work on Green's model which completely neglects the hydro-

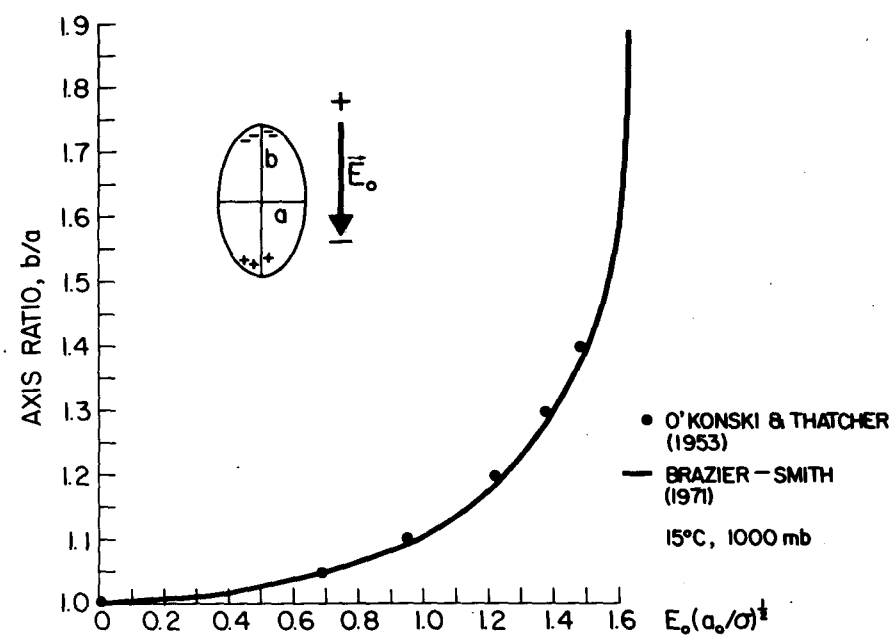


FIG. 1. Variation of theoretically predicted shape of an originally spherical water drop in air, assuming that the drop is at rest, with $X = E_0(a_0/\sigma_{wv})^{1/2}$. Results presented in literature.

dynamic flow around and inside a drop falling in air. Thus, although Green obtains ($b/a$) values for the drop shape which closely agree with those of Pitter and Pruppacher (1971), he completely misses the true drop shape; for $a_0 \gtrsim 1$ mm the shape is characterized by an increasingly flattened upstream side and an increasingly pronounced concavity in the center of this flat "bottom." As far as the effect of the electric field is concerned, Zrink's prediction of drop shape is in strong disagreement with the shapes and breakup values observed during the wind tunnel studies of Dawson and Richards (1970) and Richards and Dawson (1971), whose drops were always at terminal velocity; they were exposed to slowly increasing electric fields until breakup occurred.

Other experiments which addressed themselves to the problem of critical drop shape for breakup or onset of corona-discharge were carried out by Wilson and Taylor (1925), Nolan (1926), Macky (1931), Ausman and Brook (1967) and Griffith and Latham (1972). These authors found that for the critical condition, $X_c$ assumed values of 1.61, 1.51, 1.56 and 1.81, respectively. Unfortunately, in all of these experiments emphasis was placed on the conditions for breakup, and no quantitative determination was made regarding the variation of the shape $b/a$ with $E_0$ and with $a_0$. In addition, these experiments suffered from the fact that the drops did not reach their terminal velocity and thus did not assume their correct hydrodynamic shape. Also, the drops were forced to enter abruptly into a given external electric field region created between two electrodes, thus causing nonequilibrium conditions between the drop and its environment.

In order to eliminate some of the deficiencies of previous experiments, and in the hope of providing an experimental basis for characterizing the hydrodynamic versus electric-field effects on drop shape, we undertook new wind tunnel studies inside the UCLA cloud tunnel. This wind tunnel allows one to stably suspend drops of radii between 10 and 3000 $\mu$m freely in a low turbulence air stream of $(\bar{u}^2)^{1/2}/U$ less than 1%. Recent studies in this tunnel, however, in the *absence* of the electric fields, were carried out by Pruppacher and Beard (1970) and Pruppacher and Pitter (1971). Their studies show that drops smaller than 500 $\mu$m can be considered as spheres. On the other hand, drops larger than 500 $\mu$m radius were oblate spheroids with an increasingly flattened upstream (lower) side as the drop became larger. This deformation caused the axis ratio $b/a$ to be less than unity, with $b/a$ decreasing as $a_0$ increases. This behavior was shown to be the result of an interaction between aerodynamic, surface tension and hydrostatic forces. Thus their results demonstrate that inside a cloud an external electric field does not act—as in the above mentioned theories—on an originally *spherical* drop, but on an *oblate deformed* drop around which the aerodynamic flow changes continuously, and with it the aerodynamic forces as the electric field increases in strength. Thus the shape of a drop falling at terminal velocity and being exposed to an external electric field is the result of a complicated, continuously changing interplay between aerodynamic, surface tension, hydrostatic and electric forces.

## 3. Present experimental set up

In the present study, drops of equivalent radius 1000–3000 $\mu$m were freely suspended in the vertical air stream of the UCLA cloud tunnel (having electrically grounded metal walls). Two metal screens, mounted upstream and downstream of the drop, acted as electrodes to create a vertical electric field. The screens of 6 × 6 cm were separated by a distance of 5 cm. The upper screen was charged to a positive electric potential, the lower screen to a negative electric potential. In this way, electric fields between 1000 and 9000 volts cm$^{-1}$ were created between the screens. The drops were stably held near the axis of the flow and about halfway between the two screens. Following Smythe (1950), we computed that for our screen and wall configuration the effects of field nonuniformity were less than 1%, so we could assume that the field in the drop's vicinity was uniform and computable from $E_0 = \Delta V/D$, where $D$ is the distance between the two electrodes and $\Delta V$ is the potential difference between the two electrodes. In addition, possible corona discharge from the edges and corners of the screens were inhibited by having them tightly mounted to a plexiglass insert supported inside our metal tunnel. We would also have noticed even a slight amount of drop charging by ions released by any corona discharge, as the drop would then have moved rapidly to one or the other electrode; such movement was not observed. The drops studied were made from doubly distilled water and their size and shape were documented photographically (Fig. 2). For zero electric fields our values for the drop deformation were checked against the values of Pruppacher and Pitter (1971). All experiments were carried out in an atmosphere of about 1000 mb air pressure and about 15°C air temperature. Water drops of equivalent radius between 1000 and 3000 $\mu$m were studied by means of still photography and motion picture. In total, about 30 drops at each size were analyzed regarding their shape.

## 4. Results and discussions

The experimental results of our study are summarized in Figs. 3 to 6. Figure 3 gives the variation of the drop shape $b/a$ with the equivalent drop radius $a_0$ for various electric field strengths $E_0$. Comparison of values for $b/a$ for $E_0 = 0$ agreed favorably with the values previously obtained by Pruppacher and Pitter (1971). We note from Fig. 3 that for drops of $a_0 < 0.21$ cm and $E_0 < 4$ kV/cm the aerodynamic and hydrostatic forces dominate and cause $b/a$ to decrease with increasing $a_0$. For larger drops the electric forces pro-

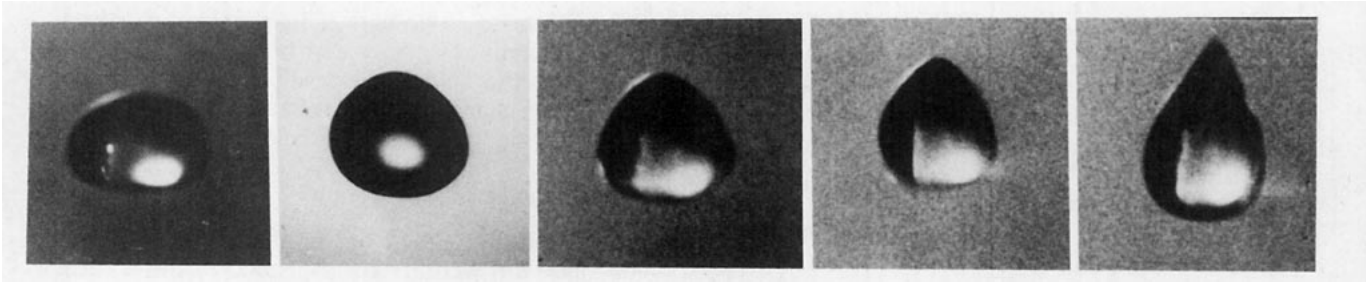

FIG. 2. Photographs from 16 mm motion picture of water drops in airstream being exposed to various external electric fields; $a_0$ = 0.23 cm. From left to right: $b/a$ = 0.725, 0.925, 0.970, 1.12, 1.52, for $E_0$ = 0, 6.75, 8.25, 9.0 and 10.0 kV/cm, respectively. Present results.

gressively take over to reverse the trend of $b/a$ with $a_0$. For electric field strengths of $E_0 > 4$ kV/cm the electric forces dominate at all drop sizes. Extrapolation shows that for drops of $a_0 < 1$ mm, external electric fields of up to 7 kV/cm have no effects on their shape.

Figure 4 gives the variation of the drop shape $b/a$ with electric field strength $E_0$ for various equivalent drop radii $a_0$. We note from this figure that the electric field strength necessary to affect the drop shape increases significantly with decreasing drop size. Thus for drops of $a_0$ = 1.7 mm the electric field strength needs to be about 5 kV/cm; it only needs to have a value of about 1.5 kV/cm if its radius is 2.3 mm. Due to this behavior a "crossover" of the plotted curves, $b/a$ versus $E_0$ for given $a_0$, occurs. The behavior can be understood if we realize that for small drops the surface tension force $2\sigma_{wv}/a_0$ dominates and requires the electrical polarization force to take over only when the electric field strength is already quite large. Large drops, on the other hand, although strongly oblately deformed due to hydrostatic and aerodynamic forces, would therefore, on first sight, need a larger electric field for deforming them into a drop of given $b/a_0$; however, they are more readily deformed by a given electric field than small drops due to the relatively small value of surface energy $\sigma_{wv}[(1/r_1) + (1/r_2)]$ where $r_1$ and $r_2$ are the principal radii of drop curvature. Therefore large drops deform with increasing $E_0$ more readily than do small drops, becoming eventually spherical and prolate.

In Figure 5 we have replotted curves (1) and (8) from Figure 4 and extrapolated them to higher values of $b/a$. We have also indicated on this figure by means of stars the values for the critical electrical field strength $E_{0c}$ at which, according to the wind tunnel observations of Richards and Dawson (1971), drops do break up under the influence of the electric field. These values

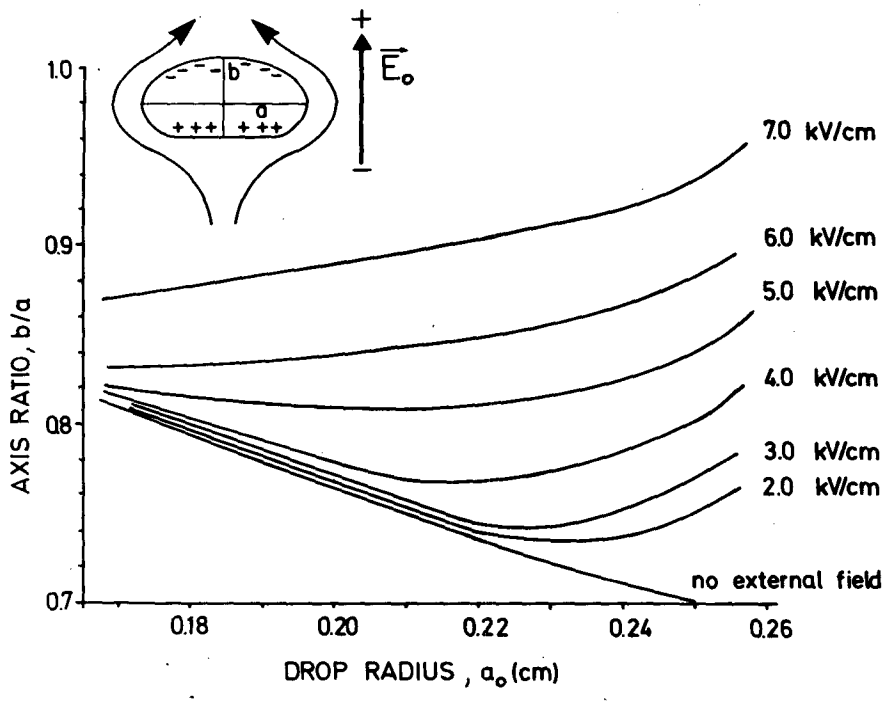


FIG. 3. Variation of the drop shape $b/a$ with drop radius $a_0$ for various external electric field strengths. Present results.

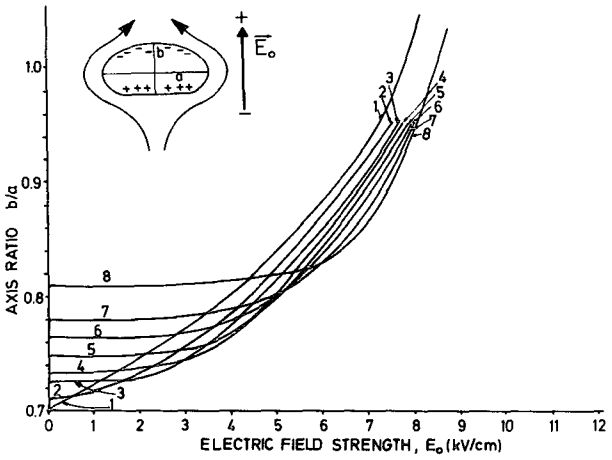


FIG. 4. Variation of the drop shape $b/a$ with electric field strength for various drop radii $a_0$; present results. Curves 1–8 refer to $a_0$ = 2.5, 2.4, 2.3, 2.2, 2.1, 2.0, 1.9 and 1.7 mm, respectively. Present data.

were retrieved from Richards and Dawson by fitting to their observational points the line $X_c = 1.6 + \alpha a_0^{\gamma}$, where $\alpha$ and $\gamma$ are regression constants with the values 7.4 and 2.6, respectively. This fit is good for the size range $0 \leqslant a_0 \leqslant 3.25$ mm. We note that our extrapolated curves are consistent with Richards' and Dawson's critical fields for drop breakup. Figure 2 illustrates such a drop breakup process. We note from this series of pictures that due to the action of an external electric field drop breakup proceeds via instabilities at the drop's *upper* (i.e. *downstream*) *side.* This break-up mechanism is in strong contrast to the breakup of single drops in the absence of an external electric field. In such a case drops of sufficiently large size develop hydrodynamic instabilities on the drop's *lower* (i.e. upstream) *side,* so that breakup proceeds from the drop's *lower side* (for reference to hydrodynamic breakup in absence of electric fields see Pruppacher and Klett, 1978, chapter 10).

In Figure 6 we have plotted the variation of the drop shape $b/a$ as a function of the nondimensional parameter $X = E_0(a_0/\sigma_{wv})^{1/2}$ for the case of the two drop sizes $a_0$ = 2.5 mm and $a_0$ = 1.7 mm. We note from this figure that $X_c$ does not have the unique features which it was thought to have based on the theoretical work with liquid *spheres* deforming in electric fields. In particular, $X_c$ appears not to be independent of $a_0$ when electric forces as well as aerodynamic and hydrostatic forces are acting on the drop. Thus, we notice that the critical value $X_c$ for breakup is close to 1.80 for a drop of $a_0$ = 2.5 mm, but is only 1.67 for a drop of 1.7 mm, according to the experiments of Richards and Dawson (1971). The critical value $X_c$ converges to about 1.60 for sufficiently small drops which are spherical in shape.

It is unfortunate that no theory is available which would predict the shape of a water drop in air under the combined effects of electric field and aerodynamic flow. Our experimental results are therefore not theoretically confirmed. However, in the light of what has been said herein, they appear to be physically reasonable, and as such are hoped to be an important guide to the radar meteorologist as well as to the cloud physicist.

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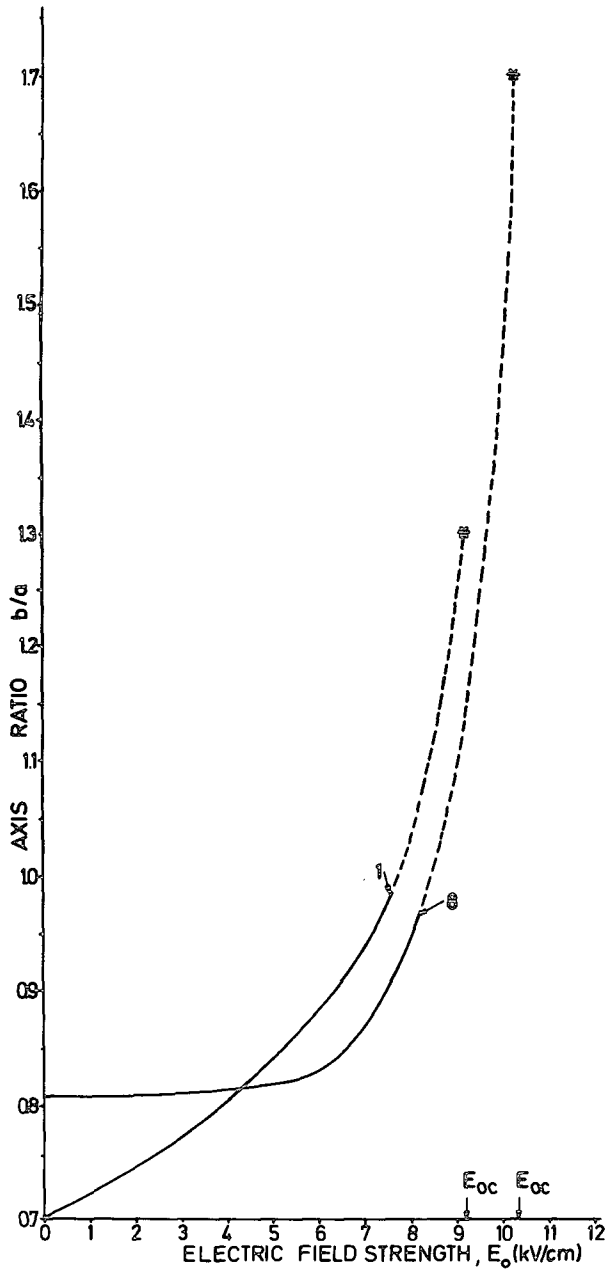


FIG. 5. Variation of the drop shape $b/a$ with electric field strengths for drops of $a_0$ = 2.5 and 1.7 mm. Continuous lines are present observed data; dashed lines are extrapolated present data; stars mark critical breakup values obtained by Richards and Dawson (1971).

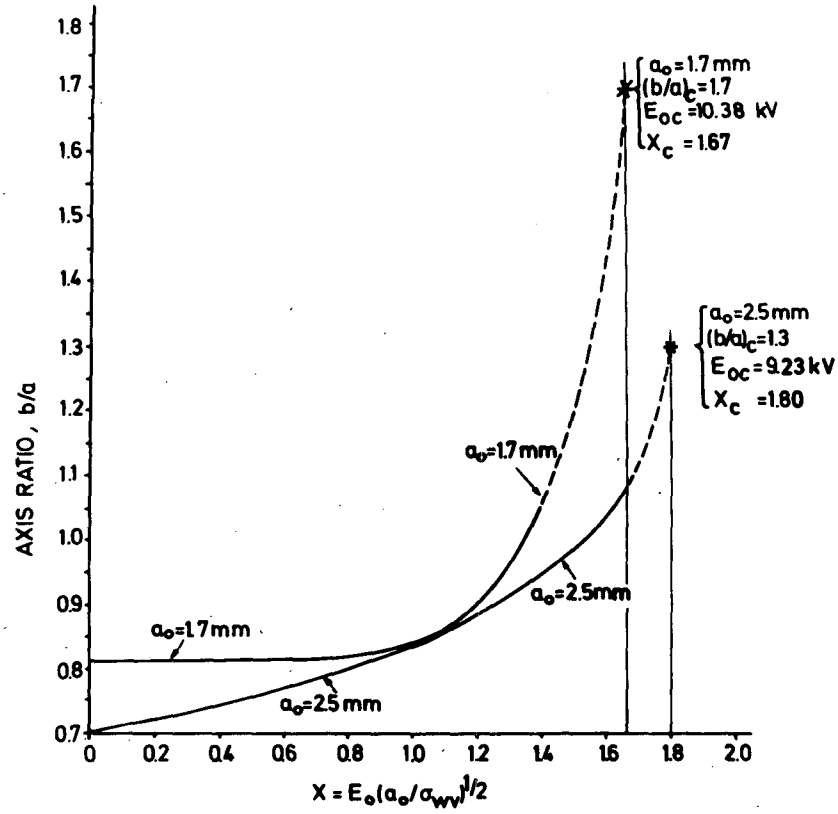


FIG. 6. As in Fig. 5 but with nondimensional parameter $X = E_0(a_0/\sigma_{wv})^{1/2}$.

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