A Numerical Study of the Effect of Electric Charges on the Efficiency with which Planar Ice Crystals Collect Supercooled Cloud Drops

J. J. MARTIN,[1] P. K. WANG[2] AND H. R. PRUPPACHER

Department of Atmospheric Sciences, University of California, Los Angeles 90024

R. L. PITTER

Department of Meteorology, University of Maryland, College Park 20742

(Manuscript received 31 March 1981, in final form 10 July 1981)

ABSTRACT

A theoretical model is presented which allows determination of the efficiency with which electrically charged, simple planar ice crystals collide with electrically charged supercooled cloud drops. The calculations are carried out for ice crystal plates of diameters between 100 and 1300 μm colliding with cloud drops of diameters between 2 and 170 μm. The electric charges Q (esu) residing on the drops and ice crystals were assumed to vary with the radius a (cm) of the drop or crystal according to $Q = qa^2$, with $0 \leq q \leq 2.0$. Our results show that the efficiency with which supercooled drops are collected by simple planar ice crystals is enhanced by electric charges, in particular, if $q > 0.8$, where $q = 0.8$ represents an electric charge still considerably below thunderstorm charge.

1. Introduction

In "mixed" clouds, where ice crystals and supercooled cloud drops are simultaneously present, the collision of ice crystals with supercooled drops contributes importantly to the formation and growth of precipitation particles. This collision mechanism, commonly termed "riming," has received considerable attention in the recent past. Thus, Locatelli and Hobbs (1974) studied the fall speed and dimensional relationship of rimed ice particles. Zikmunda and Vali (1972), Kajikawa (1977), Pflaum *et al.* (1978), and Pflaum and Pruppacher (1979) studied the fall mode and growth behavior of graupel. Ono (1969), Wilkins and Auer,[3] Hobbs *et al.*,[4] Iwai (1973), Harimaya (1975), d'Errico and Auer,[5] Kikutchi and Uyeda (1979) and, recently, Reinking (1979) made field studies to determine the efficiency with which supercooled drops are collected by ice crystals of various shapes. Pitter and Pruppacher (1974) and Pitter (1977a) attempted to simulate the collision of simple ice crystal plates by assuming the plates to behave as thin oblate spheroids of ice around which they determined the flow of air for different fall speeds from a numerical solution of the Navier-Stokes equation of motion. From the flow fields they computed the efficiency with which small water drops collide with the ice spheroids. A similar theoretical study was carried out by Schlamp *et al.* (1975) for columnar ice crystals colliding with small drops. Laboratory studies on the same subject were carried out by Sasyo (1971) and by Kajikawa (1974).

Little attention has thus far been given to the effects of electric charges on the riming process, although it is well known that in mixed clouds in which the ice particles grow by riming the ice particles and drops quite frequently carry electric charges on their surface. A first attempt to address this problem was made by Pitter (1978). Unfortunately, this first attempt was carried out for riming conditions (−18°C and 400 mb) which are not representative for riming conditions in atmospheric clouds. In the present work more appropriate conditions were used. The present work also improved the earlier expression of Pitter and Pruppacher (1974) and of Pitter (1977a) for the criterion of collision between the drop and ice crystal plate near the ice crystal tip. Moreover, the present work avoids some problems which arose

---

[1] Present affiliation: Graduate Center for Cloud Physics Research, University of Missouri, Rolla 65401.

[2] Present affiliation: Department of Meteorology, University of Wisconsin, Madison 53706.

[3] Wilkins, R. D., and A. H. Auer, 1970: Riming properties of hexagonal ice crystals. *Preprints Conf. Cloud Physics*, Fort Collins, CO, Amer. Meteor. Soc., 81–82.

[4] Hobbs, P. V., L. F. Radke, A. B. Fraser, J. D. Locatelli, J. D. Robertson, C. E. Atkinson, D. G. Farber, R. J. Weiss and R. C. Easter, 1971: Studies of winter cyclonic storms over the Cascade Mountains. Research Rep., No. 6; Dept. Atmos. Sci., University of Washington, Seattle, WA.

[5] d'Errico, R. E., and A. H. Auer, 1978: An observational study of the accretional properties of ice crystals of simple geometric shapes. *Preprints Cloud Physics Conf.*, Issaquah, Amer. Meteor. Soc., 114–121.

0022-4928/81/112462-08$06.00

due to mixing up of two oblate spheroidal coordinate systems which biased some of the results of Pitter (1977b, 1978), as pointed out by Martin *et al.* (1980).

2. Present model

In the present study we employed the trajectory model previously used by Schlamp *et al.* (1976) for describing the interaction between electrically charged drops, and by Martin *et al.* (1980) for describing the interaction between electrically charged ice crystal plates and electrically charged aerosol particles. With this model we determined the trajectory of electrically charged water drops around electrically charged ice crystal plates in air. The efficiency with which a crystal plate collides with a drop was found from the critical trajectory on which a drop just makes a grazing contact with the ice crystal plate.

The trajectory model used in the present study is based on an approximate formulation for the equation of motion of the electrically charged drop around the electrically charged ice crystal plate

$$m_d \frac{d\mathbf{v}_d}{dt} = m_d \mathbf{g}_d{}^* - \frac{6\pi\eta_a a_d}{(1+\alpha N_{Kn})_d}\left(\frac{c_D N_{\mathrm{Re}}}{24}\right)_d (\mathbf{v}_d - \mathbf{u}_c) + \mathbf{F}_{e,d} \quad (1)$$

and on an approximate form of the equation of motion of the electrically charged ice crystal around the electrically charged drop

$$m_c \frac{d\mathbf{v}_c}{dt} = m_c \mathbf{g}_c{}^* - 6\pi\eta_a a_c\left(\frac{c_D N_{\mathrm{Re}}}{24}\right)_c (\mathbf{v}_c - \mathbf{u}_d) + \mathbf{F}_{e,c} \quad (2)$$

Eqs. (1) and (2) are expressions characterizing what is known as the superposition method, according to which each hydrodynamically interacting particle is assumed to move in a flow field generated by the other falling in isolation. In Eqs. (1) and (2) m_d and m_c, $\mathbf{v}_d$ and $\mathbf{v}_c$, and a_c and a_d are the mass, velocity and radius of the drop and ice crystal plate, respectively. η_a is the dynamic viscosity of air, $\mathbf{u}_d$ and $\mathbf{u}_c$ are the velocity of the air around the drop and crystal in insolation, respectively, $\mathbf{g}^*_{d,c} = g(\rho_{d,c} - \rho_a/\rho_a)$, $\mathbf{F}_{e,d}$ and $\mathbf{F}_{e,c}$ are the electric forces caused by the presence of electric charges on the drop and crystal, $N_{Kn} = \lambda_a/a$, and $\alpha = 1.25 + 0.44\exp(-1.10N_{Kn}{}^{-1})$. The flow slip correction involving N_{Kn} was used when $a_d < 10$ μm. The drops were assumed to have the shape of perfect spheres. The shape of a simple ice crystal plate was idealized by the shape of a thin oblate spheroid of ice of axis ratio $b/a_c = 0.05$, where a_c is the major semi-axis of the spheroid (considered to be the radius of the crystal) and b is the minor semi-axis of the spheroid. Such idealization has been justified by Jayaweera and Cottis (1969) who showed experimentally that the hydrodynamic drag on a thin disk is, within a small experimental error, the same as that on a simple hexagonal plate of the same radius. A similar result was obtained by Jayaweera (1972) from a comparison of the terminal fall velocity of circular disks with simple hexagonal plates.

The velocity fields used for describing the flow field $\mathbf{u}_c$ around the falling oblate spheroids of ice of axis ratio $b/a_c = 0.05$ were those of Pitter *et al.* (1973) and Martin.[6] The velocity fields used to describe the flow field $\mathbf{u}_d$ around the falling water drops were those of Le Clair *et al.* (1972) and of S. N. Grover (1975, private communication). These flow fields were determined from a numerical solution of the complete steady state Navier-Stokes equation of motion for noncompressible flow.

The electric force, $\mathbf{F}_{e,d} = +Q_d\mathbf{E}$, acting on a drop, was assumed to result from an interaction between the charge Q_d on the drop and the electric field $\mathbf{E}$ around the ice crystal carrying a surface electric charge of magnitude Q_c. The electric field around the charged ice crystal, assumed to be a perfect electric conductor, satisfies the condition

$$\mathbf{E} = -\nabla\Phi_{e,c}, \quad (3)$$

where $\Phi_{e,c}$ is the electric potential around the crystal. Taking this potential to be zero far away from the crystal, and assuming no background electric field and no electric charges outside the ice spheroid, the electric potential satisfies Laplace's equation,

$$\nabla^2\Phi_{e,c} = 0. \quad (4)$$

In oblate spheroidal coordinates, ξ, η, described by Happel and Brenner (1965, p. 513), Eq. (4) has for the above conditions the particular solution,

$$\Phi_{e,c} = c_1 \sin^{-1}(\tanh\xi) + c_2. \quad (5)$$

The constants c_1 and c_2 may be obtained from the boundary conditions $\Phi_{e,c} = \Phi_{e,0} =$ constant for $\xi = \xi_0$, and $\Phi_{e,c} = 0$ for $\xi = \xi_\infty$, leading to

$$c_1 = -\Phi_{e,0}\left[\frac{\pi}{2} - \sin^{-1}(b/a_c)\right]^{-1} \quad (6)$$

$$c_2 = \Phi_{e,0}\left[\frac{\pi}{2} - \sin^{-1}(b/a_c)\right]^{-1}, \quad (7)$$

where b/a_c is the axis ratio of the crystal, and where $\Phi_{e,0}$ is the electric potential at the surface of the oblate spheroid of ice. From electrostatic theory

$$\Phi_{e,0} = Q_c/C, \quad (8)$$

---

[6] Martin, J. J., 1980: A numerical study of the efficiency with which aerosol particles collide with simple planar ice crystals. Ph.D. thesis, Dept. Atmos. Sci., University of California, Los Angeles, 486 pp.

where the capacitance C for a thin oblate spheroid is given by the relation (see Pruppacher and Klett, 1980)

$$C = \frac{a_c[1 - (b/a_c)^2]^{1/2}}{\sin^{-1}\{[1 - (b/a_c)^2]^{1/2}\}} . \tag{9}$$

Therefore, the electrostatic potential on any ξ surface in the fluid due to the charge on the crystal is

$$\Phi_{e,c} = \frac{Q_c}{C} \frac{\left[\frac{\pi}{2} - \sin^{-1}(\tanh\xi)\right]}{\left[\frac{\pi}{2} - \sin^{-1}(b/a_c)\right]} . \tag{10}$$

Considering Eq. (3), we then get for the electric field

$$\mathbf{E} = - \frac{Q_c \operatorname{sech}\xi}{a_c[1 - (b/a_c)^2]^{1/2}[\sinh^2\xi + \cos^2\eta]^{1/2} C \left[\frac{\pi}{2} - \sin^{-1}(b/a_c)\right]} \hat{e}_\xi \tag{11}$$

where $\hat{e}_\xi$ is the unit vector in the ξ direction, and where the terms $[1 - (b/a_c)^2]^{1/2}[\sinh^2\xi + \cos^2\eta]^{1/2}$ arise from the differentiation of Eq. (10). Therefore, the electric force, $\mathbf{F}_{e,d}$, exerted by the crystal on the drop is [together with Eq. (9)]

$$\mathbf{F}_{e,d} = + \frac{Q_c Q_d \operatorname{sech}\xi \sin^{-1}\{[1 - (b/a_c)^2]^{1/2}\}}{a_c^2[1 - (b/a_c)^2][\sinh^2\xi + \cos^2\eta]^{1/2}\left[\frac{\pi}{2} - \sin^{-1}(b/a_c)\right]} \hat{e}_\xi \tag{12}$$

The electric force $\mathbf{F}_{e,c}$ exerted by the charge on the drop onto the charged ice crystal was computed analogously to $\mathbf{F}_{e,d}$. Thus, Eq. (4) in radial coordinates has the solution

$$\Phi_{e,d} = \frac{Q_d}{4\pi r_s} , \tag{13}$$

where r_s is the center-to-center distance between the drop and ice crystal. From Eq. (3) then follows for the electric field

$$\mathbf{E} = \frac{Q_d}{4\pi r_s^2} \hat{e}_r \tag{14}$$

where $\hat{e}_r$ is the unit vector in the radial direction. Therefore, the electric force $\mathbf{F}_{e,c}$ exerted by the drop on the crystal is

$$\mathbf{F}_{e,c} = +Q_c\mathbf{E} = + \frac{Q_c Q_d}{4\pi r_s^2} \hat{e}_r . \tag{15}$$

The above formulated electric model disregards the possibility that the electric forces are strong enough to "tip" the ice crystal but rather allows the crystal to fall continuously with its broadest extension perpendicular to the fall axis. The model also disregards electric effects due to mutually induced electric charges. The former assumption has been justified by Pitter and Pruppacher (1974) who showed that for the drops and ice crystals considered, the torque exerted on the ice crystal by the drop is too small to cause the ice crystal to "tip." The latter assumption cannot be justified satisfactorily. However, from the studies of Grover and Beard (1975) for electrically charged water drops, it appears that the effect of an inductive force will not change the present computations significantly. Also, the present model is only capable of considering oblate ice spheroids with Reynolds numbers $N_{\mathrm{Re}} = 2a_c V_\infty/\nu_a$ (where V_∞ is the terminal fall velocity of the ice crystal and ν_a is the kinematic viscosity of the air) less than 100. Oblate ice spheroids with Reynolds number larger than this value exhibit shedding of eddies from their rear. For this case the Navier-Stokes equation of motion has not yet been solved since at such Reynolds numbers the flow has lost its axial symmetry and has become time dependent.

From a solution of Eqs. (1) and (2) the trajectory was computed for electrically charged drops of various sizes moving around electrically charged oblate spheroids of ice of various sizes. In turn, from these trajectories the collision efficiency

$$E = \frac{\pi y_c^2}{\pi(a_c + a_d)^2} , \tag{16}$$

was computed, where y_c, measured perpendicular to the crystal fall axis aligned along $\mathbf{g}$ and sufficiently far upstream of the crystal, is the largest horizontal offset a drop can have from the ice crystal fall axis and still collide with the spheroid.

For the case of an *electrically uncharged* drop and ice crystal it was insured that the crystal and drop had an initial vertical separation which was sufficiently large that any additional increase in vertical separation changed the collision efficiency by <0.1%, insuring that beyond the chosen vertical separation any variation in hydrodynamic interaction had a negligible effect on the collision efficiency.

For the case of *electrically charged* drops and ice crystals two cases were considered. In the *first case* (results in Fig. 1) the drops and ice crystals had an initial vertical separation *equal* to that chosen for the uncharged particles. In the *second case* (results in Fig. 2) the drop and the ice crystal had an initial vertical separation which was sufficiently large to insure that by changes in electrical as well as hydrodynamic interactions at larger vertical separation changed the collision efficiency by <0.1%. The *second case* represents the situation as it is found in an atmospheric cloud free of turbulence. The *first* case, on the other hand, pertains to a cloud in which turbulent eddies bring cloud drops and ice crystals into close proximity. This latter case, however, only represents a lower bound to the collision efficiency in a turbulent cloud, since the effects of turbulent accelerations between the drops on the collision efficiency have not been considered.

The numerical method used for computing the drop trajectories and collision efficiencies followed essentially the method of Pitter and Pruppacher (1974) with some additions and refinements discussed in detail by Martin.[6]

The oblate spheroids of ice studied had an axis ratio of $(b/a_c) = 0.05$, and Reynolds numbers $N_{\mathrm{Re},c}$ and radii a_c listed in Table 1. For the oblate ice spheroids listed in Table 1 the terminal fall velocity in air was computed from the hydrodynamic drag on these bodies according to the method given by Pitter and Pruppacher (1974). The latter, in turn, was determined according to the method of Pitter *et al.* (1973) from the flow fields around these bodies. In Table 1 comparison is made between the theoretically predicted fall velocity, available for 700 mb and −10°C, and the fall velocity experimentally measured by Kajikawa (1972) for 1000 mb and −10°C. Considering the difference in pressure, the two data sets appear to be in good agreement and thus justify further our idealization of simple hexagonal ice crystal plates by thin oblate spheroids of ice.

A literature search (Magono and Kikuchi, 1961; Isono *et al.*, 1966; Burrows and Hobbs, 1970; Kikuchi, 1973; and Magono and Iwabuchi (1979) reveal that quantitative data on the electric charge on platelike ice crystals are scarce. The available data, however, suggest some upper bounds from which it was determined that the surface charge on an ice crystal plate in *strongly electrified* clouds may be represented by

$$|Q_c| = |q_c| a_c^2 \tag{17}$$

with $|q_c| = 2.0$, and with Q_c in e.s.u. and a_c in centimeters. An analogous law was shown by Pruppacher and Klett (1980) to hold for cloud drops. Thus, we assumed for strongly electrified clouds

$$|Q_d| = |q_d| a_d^2 \tag{18}$$

with $|q_d| = 2.0$, and with Q_d in e.s.u. and a_d in cm.

It also appears from the field observations cited above that platelike ice crystals are predominantly negatively charged. In contrast, in partially glaciated clouds, the drops may carry positive as well as negative electric charge. However, in the present study we considered only collisional interactions between an ice crystal and a drop of *opposite* charge sign. Due to the strongly opposing electric forces, it is obvious that the efficiency with which a drop and ice crystal of *alike* charge sign collide is very small.

3. Results and discussion

In Fig. 1 we have summarized our results for the efficiency with which uncharged drops and uncharged simple ice crystal plates collide with each other

TABLE 1. Reynolds numbers, terminal fall velocity and electrical charge for simple ice crystal plates.

		Terminal fall velocity of ice crystal (cm s^{-1})		
Reynolds number $N_{\mathrm{Re},c}$	Crystal radius a_c (μm)	Computed for oblate spheroids of ice of $(b/a) = 0.05$, 700 mb, −10°C	Observed for simple hexagonal ice crystals (Kajikawa, 1972) 1000 mb, −10°C, ±3 cm s^{-1}	Electric charge Q_c (esu) on the ice crystal for $Q_c = q_c a_c^2$ with $q_c = -2.0$
0.1	51	1.8	2	-5.2×10^{-5}
0.5	88	5.1	5	-1.5×10^{-4}
1.0	113	8.0	8	-2.5×10^{-4}
2.0	147	12.3	12	-4.3×10^{-4}
2.5	160	14.1	14	-5.2×10^{-4}
4	194	18.6	18	-7.5×10^{-4}
5	213	21.2	21	-9.1×10^{-4}
10	289	31.1	30	-1.7×10^{-3}
20	404	44.6	42	-3.3×10^{-3}
50	639	70.4	65	-8.2×10^{-3}

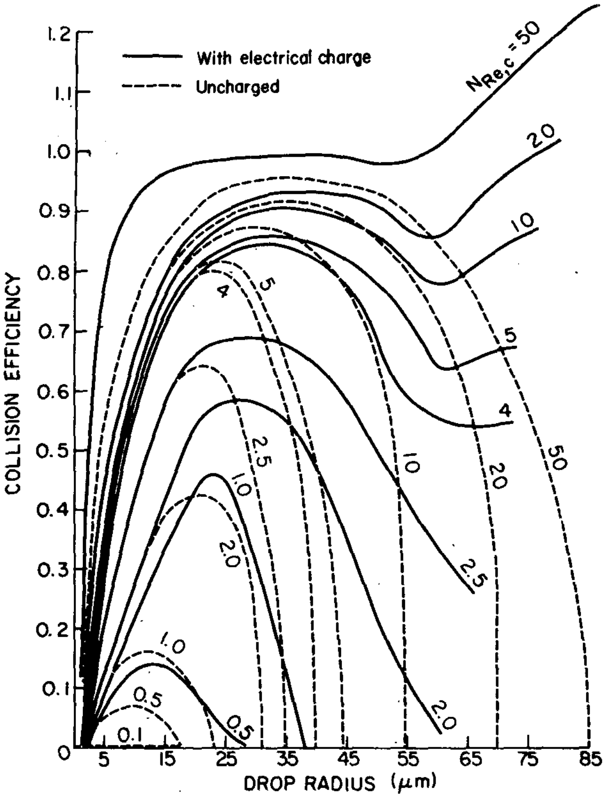

FIG. 1. Efficiency with which a simple hexagonal ice plate of given Reynolds number (see Table 1) collides with a supercooled drop of given size, for the case of electrically charged and uncharged ice crystals and drops; for 700 mb and −10°C; for $Q_c = q_c a_c^2$, $Q_d = q_d a_d^2$, with $q_c = q_d = 2.0$; and for initial vertical separations specified in the text.

(dashed line), and our results for the efficiency with which electrically charged simple ice crystal plates collide with oppositely charged drops (solid line), in air of 700 mb and −10°C, and of the same vertical separation as the uncharged drops and crystals. Both efficiencies are plotted as a function of a drop size for various crystal sizes.

We note from Fig. 1 that for electrically uncharged drops and crystals the collision efficiency rapidly increases with increasing drop size up to a broad maximum beyond which the efficiency rapidly decreases to zero. The above mentioned behavior of small drops is, of course, a result of the fact that very small drops have little inertia and are, by means of hydrodynamic forces, swept around the crystal rather than forced to collide with it. This result has been verified by the field observations of Wilkins and Auer,[3] Harimaya (1975) and Kikuchi and Uyeda (1979), who found that drops smaller than $a_d \leqslant 5$ μm are rarely found on rimed ice crystal plates collected in natural clouds. On the other hand, large drops rapidly approach the terminal fall velocity of the ice crystal plates considered here and therefore will elude capture by them. Unfortunately, no comparison can be made between our theoretically predicted collision efficiency cutoff for large drops and that observed from field observations. This is because the clouds studied by Wilkins and Auer, Harimaya, and Kikuchi and Uyeda contained either none or only a few such large drops.

We further note from Fig. 1 that there exists a minimum ice crystal size below which no drops were collected. The present theoretical study suggests that this cutoff occurs for ice crystal plates of radii between 51 and 88 μm (i.e., $102 \leqslant D_c \leqslant 176$ μm, where D_c is the ice crystal diameter). The results are in good agreement with the most recent, extensive field observations of Reinking (1979), summarized in Table 2. Fig. 1 also shows that the larger the riming ice crystal, the wider the drop size spectrum of the drops collected. This result also agrees with the field observations mentioned above.

A comparison of the present results for uncharged drops and ice crystals with the results previously computed by Pitter and Pruppacher (1974) and Pitter (1977) show that the previous results considerably underestimate the efficiency with which ice crystal plates, in particular the smaller plates, capture supercooled drops. This underestimate has to be attributed to an insufficiently accurate criterion for collision. Unfortunately, these inaccuracies were not immediately discovered since the minimum ice crystal size for onset of riming theoretically predicted by those previous computations agreed quite well with the field observations of Ono (1969), Wilkins and Auer[3] and Hobbs *et al.*[4] However, the recent, much more extensive field studies on the riming onset of ice crystals by Reinking revealed that riming may begin on ice crystals smaller than those observed in the earlier field studies. The agreement between the present revised theoretical computations and Reinking's observations strongly supports this notion.

We turn now to the results shown in Fig. 1 for

TABLE 2. Critical ice crystal dimension (μm) for riming (from Reinking, 1979).

	Major ice crystal dimension		Ice crystal width	
Ice crystal type	Average min. size at onset	Average size for 25% coverage	Average min. size at onset	Average size for 25% coverage
Needles	220	1300	31	134
Sheaths	185	720	36	110
Columns	125	450	30	155
Plates	150	None	—	—
Branched planar	240	820	—	—
Radiating planar	320	700	—	—

electrically charged drops and ice crystals subject to vertical separations as found in turbulent clouds. We note from this figure that for ice crystals of $a_c \leqslant 404$ μm the charge has little effect on the collision efficiency if the drops are sufficiently small. For $160 \leqslant a_c \leqslant 404$ μm, this critical drop radius is ~20 μm. For smaller ice crystals the critical drop size decreases with decreasing crystal size, reaching a drop radius of ~4 μm for $a_c = 88$ μm. For drops larger than the critical size the electric charge increasingly enhances the collision efficiency. For ice crystals of $a_c \gtrsim 194$ μm the collision efficiency undergoes a maximum with increasing drop size, reaching a minimum with further increase in drop size. Beyond this minimum the efficiency increases again with further increase in drop size. In order to understand this behavior we must recall that the results given in Fig. 1 are derived for the case that the initial separation assumed to exist between the electrically charged drop and crystal was the same as that between uncharged drops. This assumption implied that the vertical separation between a drop and crystal guaranteed negligible initial hydrodynamic effects on the collision efficiency but on the other hand did allow initial electrical interactions between drop and crystal. These electric interactions were sufficiently strong that even larger drops cannot escape being captured by ice crystal plates if $a_c \gtrsim 200$ μm.

This situation changes drastically if we consider the situation in a turbulence-free cloud and allow the initial separation between the drop and ice crystal to be such that at any larger vertical separation the effects of the electrical interaction *as well as* hydrodynamic interaction change the collision efficiency by <0.1% (solid line in Fig. 2). Under these conditions the effects of the electrical charges to enhance the collision efficiency is limited to a much narrower range of drop sizes than under the conditions assumed for Fig. 1, and drops of terminal velocity similar to or larger than that of a crystal now can escape capture by the crystal despite the presence of the charge.

In Fig. 3 we plotted the effect of increasing electric charge on the collision efficiency of three drop–ice crystal pairs, subject to the initial vertical separation specified for getting the results in Fig. 1. It can be seen that the effects of electric charge on the collision efficiency become important for $q_c = q_d \gtrsim 0.8$, which suggests that the growth rate of ice crystals by riming becomes electrically enhanced considerably before a storm reaches a mature thunderstorm stage ($q = 2.0$). Fig. 3 also shows that the relative enhancement of the collision efficiency by electrical charge decreases with increasing size of the colliding particles involved.

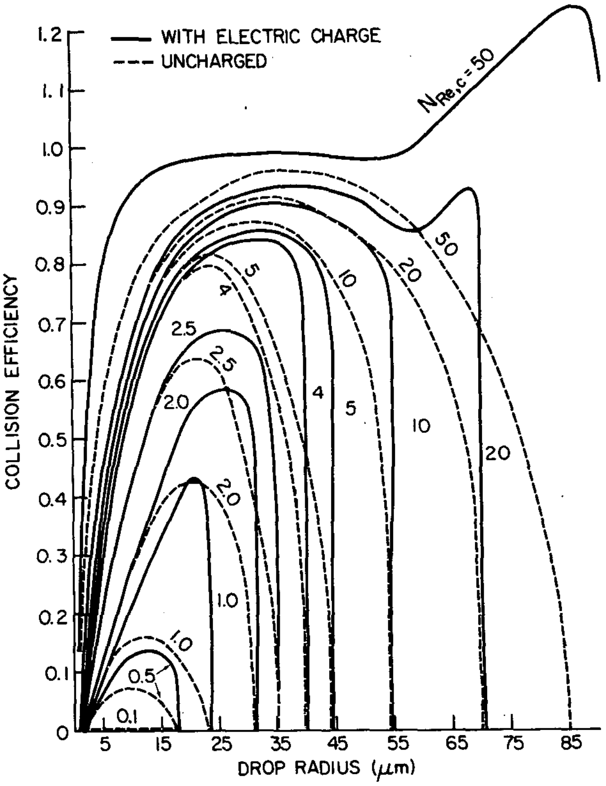

FIG. 2. As in Fig. 1, except for the larger initial vertical separations specified in the text.

In contrast to the studies of Martin *et al.* (1980), who showed that sufficiently small, electrically charged aerosol particles may be captured on the *rear side* of electrically charged ice crystal plates, the present study indicates that, for the drops and crystal sizes considered, no rear capture of drops takes place.

We are quite aware of the fact that both electrical interaction models considered here (see Figs. 1 and 2) are just two special cases of all the cases possible in atmospheric clouds. Thus, through the action of turbulent mixing, electrically charged drops may come into collision position with respect to an electrically charged ice crystal at a variety of vertical separations. Two of these vertical separations which we felt have special physical significance were considered in this article.

In closing, it must be emphasized that our results only apply to simple hexagonal ice crystal plates. Other planar crystal shapes, such as branched ice crystals, stallars, etc., most likely exhibit different efficiencies. This fact is brought out by Table 2 which shows that the crystal size for the onset of riming is considerably larger for a dendritic crystal than the size for riming onset on simple ice plates. Unfortunately, our model cannot accommodate the more complicated planar ice crystal shapes. Work on columnar ice crystals is in progress, however.

It must also be pointed out that the effects of a

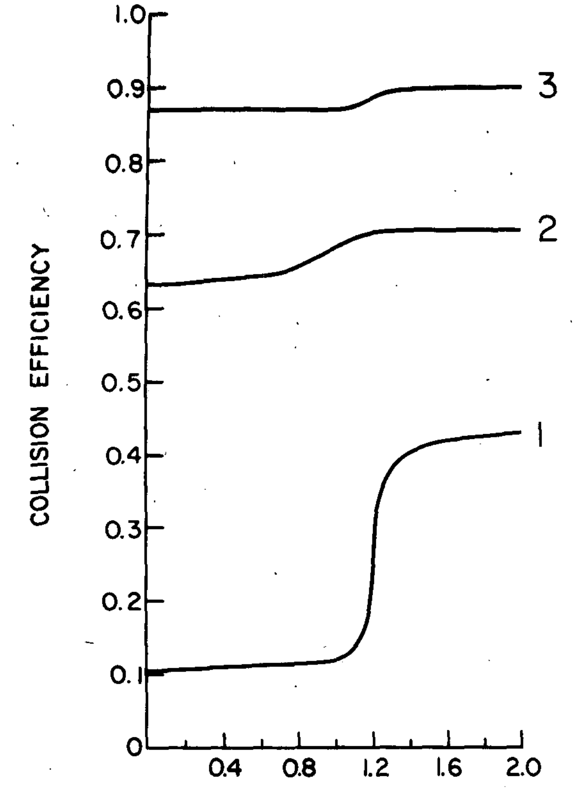

FIG. 3. Efficiency with which a simple hexagonal ice plate of given Reynolds number collides with supercooled drops of given size as a function of electrical charge $Q_c = q_c a_c^2$ and $Q_d = q_d a_d^2$, with $|q_c| = |q_d| = |q|$, varying from $|q| = 0$ to $|q| = 2.0$; for 1: $a_c = 113$, $a_d = 19.1$ μm; 2: $a_c = 289$, $a_d = 30.8$ μm; 3: $a_c = 404$, $a_d = 60.4$ μm; for the initial vertical separations of Fig. 1.

simultaneously present external electric field were not considered in the present work. As shown by Schlamp *et al.* (1976, 1979), an external electric field may increase or decrease the collision efficiency between cloud drops, depending strongly on the orientation of the electric field and on the electric sign of the collector and the collected drops. The effects of a simultaneously present external electric field will be taken up in a later study.

It finally should be pointed out that the air in atmospheric clouds is often highly turbulent. The present study, however, does not consider the effects of turbulence-induced accelerations or random paths of the drops and ice crystals.

It would be highly desirable to have available some experimental verification of the results of our theoretical study. Unfortunately, the two laboratory studies available (Sasyo, 1971; Kajikawa, 1974) are quite crude and apply to ice crystal *models* rather than real ice crystals, and to model sizes of $D_c \geqslant 5000$ μm (where D_c is the crystal diameter), larger than those considered presently. In addition, the experiments of Sasyo were carried out for flow velocities of the viscous medium which did not correspond to the terminal velocity of an ice crystal of chosen size. Therefore, except for the field verification of our theoretically predicted onset for riming on simple ice plates, our theoretical collision efficiencies for $51 \leqslant a_c \leqslant 639$ μm and $1 \leqslant a_d \leqslant 85$ μm still await experimental verification.

Acknowledgments. The authors acknowledge the Atmospheric Sciences Section of the National Science Foundation which supported the present work under Grant ATM-78-10817 and ATM 79-09750. One of us (J.J.M.) also acknowledges the University of Missouri at Rolla for providing computer funds to the Graduate Center for Atmospheric Research (Prof. J. Kassner, Director) where portions of this work were carried out.

REFERENCES

Burrows, D. A., and Hobbs, P. V., 1970: Electrical charges on snow particles. *J. Geophys. Res.*, **75**, 4499–4505.

Grover, S. N., and K. V. Beard, 1975: A numerical determination of the efficiency with which electrically charged cloud drops and small raindrops collide with electrically charged spherical particles of various densities. *J. Atmos. Sci.*, **32**, 2156–2165.

Happel, J., and H. Brenner, 1965: *Low Reynolds Number Hydrodynamics*. Prentice Hall, 553 pp.

Harimaya, T., 1975: The riming properties of snow crystals. *J. Meteor. Soc. Japan*, **53**, 384–392.

Isono, K., M. Komobayashi and T. Takahashi, 1966: A present study of solid precipitation from convective clouds over the sea, Part III. *J. Meteor. Soc. Japan*, **44**, 227–233.

Iwai, K., 1973: On the characteristic features of snow crystals developed along *c*-axis. *J. Meteor. Soc. Japan*, **51**, 458–466.

Jayaweera, K. O. L. F., 1972: An equivalent disc for calculating the terminal velocities of plate-like ice crystals. *J. Atmos. Sci.*, **29**, 596–598.

——, and R. E. Cottis, 1969: Fall velocities of plate-like and columnar crystals. *Quart. J. Roy. Meteor. Soc.*, **95**, 703–709.

Kajikawa, M., 1972: Measurement of falling velocities of individual snow crystals. *J. Meteor. Soc. Japan*, **30**, 577–583.

——, 1974: On the collection efficiency of snow crystals for cloud droplets. *J. Meteor. Soc. Japan*, **52**, 328–336.

——, 1977: Observation of the fall attitudes of cone-like graupel particles. *Mem. Fac. Educ., Akita Univ.*, Nat. Sci. Ser. No. 27, 78–85.

Kikuchi, K., 1973: On the polarity of the electric charges on snow crystals of various shapes. *J. Meteor. Soc. Japan*, **51**, 337–345.

——, and H. Uyeda, 1979: Cloud droplets and raindrops collected and frozen on natural snow crystals. *J. Meteor. Soc. Japan*, **57**, 273–280.

Le Clair, B. P., A. E. Hamielec, H. R. Pruppacher and W. D. Hall, 1972: A theoretical and experimental study of the internal circulation in water drops falling at terminal velocity in air. *J. Atmos. Sci.*, **29**, 728–740.

Locatelli, J. D., and P. V. Hobbs, 1974: Fall speeds and masses of solid precipitation particles. *J. Geophys. Res.*, **79**, 2185–2197.

Magono, C., and K. Kikuchi, 1961: On the electrical charge of relatively large natural cloud particles. *J. Meteor. Soc. Japan*, **39**, 258–268.

——, and T. Iwabuchi, 1979: The electric charge on individual ice crystals. *J. Meteor. Soc. Japan,* **57,** 207–212.

Martin, J. J., P. K. Wang and H. R. Pruppacher, 1980: A theoretical study of the effect of electric charges on the efficiency with which aerosol particles are collected by ice crystal plates. *J. Colloid Interface Sci.,* **78,** 44–56.

Ono, A., 1969: The shape and riming properties of the ice crystals in natural clouds. *J. Atmos. Sci.,* **26,** 138–147.

Pflaum, J. C., J. J. Martin and H. R. Pruppacher, 1978: A wind tunnel investigation of the hydrodynamic behavior of growing, freely falling graupel. *Quart. J. Roy. Meteor. Soc.,* **104,** 179–187.

——, and H. R. Pruppacher, 1979: A wind tunnel investigation of the growth of graupel initiated from frozen drops. *J. Atmos. Sci.,* **36,** 680–689.

Pitter, R. L., 1977a: A reexamination of riming on thin ice plates. *J. Atmos. Sci.,* **34,** 684–685.

——, 1977b: Scavenging efficiency of electrically charged thin ice plates and spheroidal aerosol particles. *J. Atmos. Sci.,* **34,** 1797–1800.

——, 1978: Influence of electrostatic forces on riming by thin ice plates. *J. Meteor. Soc. Japan,* **56,** 523–526.

——, and H. R. Pruppacher, 1974: A numerical investigation of collision efficiencies of simple ice plates colliding with supercooled water drops. *J. Atmos. Sci.,* **31,** 551–559.

——, —— and A. E. Hamielec, 1973: A numerical study of the viscous flow past a thin oblate spheroid at low and intermediate Reynolds numbers. *J. Atmos. Sci.,* **30,** 125–134.

Pruppacher, H. R., and J. D. Klett, 1980: *Microphysics of Clouds,* 2nd Printing, D. Reidel, 714 pp.

Reinking, R., 1979: The onset and early growth of snow crystals by accretion of droplets. *J. Atmos. Sci.,* **36,** 870–881.

Sasyo, Y., 1971: Study of the formation of precipitation by the aggregation of snow particles and the accretion of cloud droplets on snowflakes. *Pap. Meteor. Geophys.,* **22,** 69–142.

Schlamp, R. J., H. R. Pruppacher and A. E. Hamielec, 1975: A numerical investigation of the efficiency with which simple columnar ice crystals collide with supercooled water drops. *J. Atmos. Sci.,* **32,** 2330–2337.

——, S. N. Grover, H. R. Pruppacher and A. E. Hamielec, 1976: A numerical investigation of the effect of electric charges and vertical external electric fields on the collision efficiency. *J. Atmos. Sci.,* **33,** 1747–1755.

——, ——, —— and ——, 1979: A numerical investigation of the effect of electric charges and vertical external electric fields on the collision efficiency of cloud drops, part III. *J. Atmos. Sci.,* **36,** 339–349.

Zikmunda, J., and G. Vali, 1972: Fall patterns and fall velocity of rimed ice crystals. *J. Atmos. Sci.,* **29,** 1334–1347.